文章编号:2095 -476X(2014)06 -0103 -06

连续红利支付的跳 - 扩散模型下幂期权的定价

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摘要:在假设股票价格服从带非齐次 Poisson 跳 - 扩散过程且在连续时间支付红利的情况下,建立了股票价格行为模型,同时应用保险精算法给出一类奇异期权——欧式幂期权——看涨和看跌两种情形的定价公式,以推广 Merton 关于期权定价的结果. 得到的结果优于无红利支付的情况,使该定价公式更接近市场实际情况.

关键词:跳 - 扩散过程;幂期权;连续红利

中图分类号:O211.6;F830.91 **文献标志码**:A **DOI**:10.3969/j.issn.2095 -476X.2014.06.023

Power option pricing of the continuous dividend payment model with jump-diffusion

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Abstract: Assuming that the stock company pays dividend continuously and the dividend was related with the price of the stock in the time that the stock company pays dividend, and the pricing process was jump-diffusion process, the jump process was Poisson process, the stock pricing model was established. And it gave the European call power option and the European put power option pricing model using insurance actuary pricing. The result of Merton on European option pricing was generalized. It was superior to no-dividend payment and it was more closed to the actual market situation.

Key words: jump-diffusion process; power option; continuous dividend

0 引言

期权定价问题一直是金融数学的研究热点之一,F. Black 等[1] 给出了著名的 B-S 公式,但所提出的几何布朗运动与市场实际情况有差距. 接下来很多学者致力于对股票价格发生波动的规律进行研究,提出了 Ito 过程和随机点过程混合模型、随机利率模型、一般标志过程模型等. 随后 R. Merton[2] 在 B-S 模型基础上提出股票价格服从跳 - 扩散过程. M. Bladt 等[3] 提出一种全新的期权定价方式——保险精算法. 闫海峰等[4] 假设股票价格变化过程遵循带有非齐次 Poisson 跳的扩散过程,利用保险精算法给出欧式期权定价公式及其推广公式. 文献[5 -7]给出有红利支付的一般期权的定价,文献[8 -9]分别给出无红利支付下的不同情况幂期权的定价. 在实际市场中,股票价格不仅会发生跳跃,而且还会进行红利支付. 因此,本文在假设股票价格过程服从带非齐次 Poisson 跳的扩散过程且股票进行连续红利支付的基础上,建立股票价格的市场模型,推导出欧式幂期权看涨和看跌两种情况下的定价公式,以推广 Merton 的跳 - 扩散模型的结果.

收稿日期:2014 -06 -16

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1 股票价格行为模型

考虑一个由两项资产($S(t)$,$B(t)$)构成的金融市场,其中 $S(t)$ 表示在$[0,T]$时间内连续交易的风险资产(股票)的价格,$B(t)$ 为无风险资产(债券)的价格,其中风险资产(股票)的价格过程$\{S(t);t\geqslant 0\}$满足

$$\mathrm{d}S(t)=S(t)[(u(t)-\lambda(t)\theta-q(t))\mathrm{d}t+\sigma(t)\mathrm{d}W(t)+\varphi\mathrm{d}N(t)]\qquad S(0)=s \quad ①$$

另一种是无风险资产,其价格过程$\{B(t);t\geqslant 0\}$满足 $\mathrm{d}B(t)=B(t)r(t)\mathrm{d}t,B(0)=1.$

根据 Ito 引理,随机微分方程①的解为

$$S(t)=S(0)\exp\left\{\int_0^t[\mu(s)-q(s)-\lambda(s)\theta-\frac{1}{2}\sigma^2(s)]\mathrm{d}s+\int_0^t\sigma(s)\mathrm{d}W(s)+\sum_{i=0}^{N(t)}\ln^{(1+\Phi_i)}\right\}$$

由幂期权的定义可知,将到期日的价值用标的资产价格的某个指数幂与执行价格相比较,该期权的收益函数可看成是由 $S^{\partial}(t)(\partial>0)$ 与执行价格 K 决定的. 因此

$$S^{\partial}(t)=s^{\partial}\exp\left\{\int_0^t\partial[\mu(s)-q(s)-\lambda(s)\theta-\frac{1}{2}\sigma^2(s)]\mathrm{d}s+\int_0^t\partial\sigma(s)\mathrm{d}W(s)+\partial\sum_{i=0}^{N(t)}\ln^{(1+\Phi_i)}\right\}$$

其中,$\mu(t)$,$\sigma(t)$ 分别表示股票瞬时期望收益率和瞬时价格波动率;$r(t)$ 是瞬时无风险利率;$\mu(t)$,$\sigma(t)$,$r(t)>0$ 且均为可积函数;$W(t)$ 表示定义在完备概率空间(Ω,F,P)上的标准布朗运动;$N(t)$ 表示在给定时间内跳跃的次数,是与 $W(t)$ 独立参数为 $\lambda(t)$ 的非齐次 Poisson 过程,$\lambda(t)\geqslant 0$ 且为可积函数;Φ 为股票每次跳跃的高度,$\Phi_i(i=1,2,\cdots,N(t))$ 与 $N(t)$ 独立,$\ln(1+\Phi)\sim N\left(\ln(1+\theta_i)-\frac{1}{2}\sigma_J^2,\sigma_J^2\right)$,$\sigma_J^2$ 为 $\ln(1+\theta)$ 的方差,θ 是 Φ 的无条件期望,表示股票价格由 Poisson 跳会带来的平均增长率;$q(t)$ 为连续红利率.

2 幂期权保险精算法定价

利用鞅方法求解,其过程过于繁琐,根据保险精算方法的思路,给出欧式幂期权的公平保费价格,可避开寻找等价鞅测度的困难.

引理 1 幂期权在现在时刻的价值定义为

$$C(K,T)=E\Big[\Big\{\exp\Big\{-\int_0^T\mu(s)\mathrm{d}s\Big\}S^{\partial}(T)-\exp\Big\{-\int_0^T r(s)\mathrm{d}s\Big\}K\Big\}\cdot$$
$$I\Big\{\exp\Big\{-\int_0^T\mu(s)\mathrm{d}s\Big\}S^{\partial}(T)>\exp\Big\{-\int_0^T r(s)\mathrm{d}s\Big\}K\Big\}\Big]$$

$$P(K,T)=E\Big[\Big\{\exp\Big\{-\int_0^T r(s)\mathrm{d}s\Big\}K-\exp\Big\{-\int_0^T\mu(s)\mathrm{d}s\Big\}S^{\partial}(T)\Big\}\cdot$$
$$I\Big\{\exp-\int_0^T\mu(s)\mathrm{d}s\Big\}S^{\partial}(T)>\exp\Big\{-\int_0^T r(s)\mathrm{d}s\Big\}K\Big\}\Big]$$

如果利用保险精算法,则是将风险资产按期望收益率折现,将无风险资产按无风险利率折现,这样我们就可以得到上述定义.

为了方便起见,令 $C(K,T)=C_1+C_2$,其中

$$C_1=E\Big[\exp\Big\{-\int_0^T\mu(s)\mathrm{d}s\Big\}S^{\partial}(T)I\Big\{\exp\Big\{-\int_0^T\mu(s)\mathrm{d}s\Big\}S^{\partial}(T)>\exp\Big\{-\int_0^T r(s)\mathrm{d}s\Big\}K\Big\}\Big]$$

$$C_2=E\Big[\exp\Big\{-\int_0^T r(s)\mathrm{d}s\Big\}k\cdot I\Big\{\exp\Big\{-\int_0^T\mu(s)\mathrm{d}s\Big\}S^{\partial}(T)>\exp\Big\{-\int_0^T r(s)\mathrm{d}s\Big\}K\Big\}\Big]$$

定理 1 股票价格服从上述模型要求,欧式看涨幂期权的保险精算法定价公式为

$$C(K,T)=s^{\partial}\exp\left\{\int_0^t\left[(\partial-1)\mu(s)-\partial q(s)-\partial\lambda(s)\theta-\frac{\partial(\partial-1)}{2}\sigma^2(s)\right]\mathrm{d}s\right\}\prod_1-k\exp\left\{-\int_0^T r(s)\mathrm{d}s\right\}\prod_2$$

其中

$$\prod_1=\sum_{i=0}^{+\infty}\frac{\exp\left\{-\int_0^T\lambda(s)\mathrm{d}s\right\}\left(\int_0^T\lambda(s)\mathrm{d}s\right)^n}{n!}\cdot(1+\theta)^{n\partial}\exp\left\{\frac{\partial^2}{2}\int_0^T\sigma^2(s)\mathrm{d}s+\frac{n\partial(\partial-1)}{2}\sigma_J^2\right\}N(d_n)$$

$$\prod_2 = \sum_{i=0}^{+\infty} \frac{\exp\left\{-\int_0^T \lambda(s)\,\mathrm{d}s\right\}\left(\int_0^T \lambda(s)\,\mathrm{d}s\right)^n}{n!} N\left(d_n - \sqrt{\partial^2\left(\int_0^T \sigma^2(s)\,\mathrm{d}s + n\sigma_J^{\ 2}\right)}\right)$$

$$d_n = \frac{\ln\dfrac{(1+\theta)^{n\partial}}{k} + n\partial\left(\partial - \dfrac{1}{2}\right)\sigma_J^{\ 2} + \int_0^T\left[(\partial-1)\mu(s) - \partial q(s) - \partial\lambda(s)\theta + \dfrac{\partial(2\partial-1)}{2}\sigma^2(s) + r(s)\right]\mathrm{d}s}{\sqrt{2\partial^2\left(\int_0^T \sigma^2(s)\,\mathrm{d}s + n\sigma_J^{\ 2}\right)}}$$

证明 由 $\Phi_i(i = 1,2,\cdots,N(t))$ 是相互独立且同分布的,即

$$E\left[\prod_{i=0}^{N(t)} 1 + \Phi_i\right] = E\left[E\left[\prod_{i=0}^{N(t)} 1 + \Phi_i \,\middle|\, N(t)\right]\right] = \exp\left\{\int_0^T \theta\lambda(s)\,\mathrm{d}s\right\}$$

再由$\int_0^T \sigma(s)\,\mathrm{d}W(s) \sim N\left(0,\int\sigma^2(s)\,\mathrm{d}s\right)$可得 $ES(T) = s\exp\left\{\int_0^T \mu(s)\,\mathrm{d}s\right\}$,则 $ES^{\partial}(T) = s^{\partial}\exp\left\{\int_0^T \mu(s)\,\mathrm{d}s\right\}$,所以 $\exp\left\{\int_0^T \beta(s)\,\mathrm{d}s\right\} = \dfrac{ES^{\partial}(T)}{s^{\partial}} = \exp\left\{\int_0^T \mu(s)\,\mathrm{d}s\right\}$.

又因为 $\exp\left\{-\int_0^T \mu(s)\,\mathrm{d}s\right\}S^{\partial}(T) > \exp\left\{-\int_0^T r(s)\,\mathrm{d}s\right\}k$ 等价于

$$\partial\left[\int_0^T \sigma(s)\,\mathrm{d}W(s) + \sum_{i=0}^{N(t)} \ln(1+\Phi_i)\right] >$$

$$\ln\frac{k}{s^{\partial}} - \int_0^T\left[(\partial-1)\mu(s) - \partial q(s) - \partial\lambda(s)\theta - \frac{\partial}{2}\sigma^2(s) + r(s)\right]\mathrm{d}s \triangleq d$$

对于给定的 n,由于

$$\partial\left[\int_0^T \sigma(s)\,\mathrm{d}W(s) + \sum_{i=0}^{N(T)} \ln(1+\Phi_i)\right] \triangleq x \sim N\left(\partial n\left[\ln(1+\theta) - \frac{1}{2}\sigma_J^{\ 2}\right], \partial^2\left[\int_0^T \sigma^2(s)\,\mathrm{d}s + n\sigma_J^{\ 2}\right]\right)$$

则

$$E[e^x I\{x > d\}] =$$

$$E\left[\exp\left\{\partial\left[\int_0^T \sigma(s)\,\mathrm{d}W(s) + \sum_{i=0}^{N(T)} \ln(1+\Phi_i)\right]\right\} I\left\{\partial\left[\int_0^T \sigma(s)\,\mathrm{d}W(s) + \sum_{i=0}^{N(T)} \ln(1+\Phi_i)\right] > d\right\}\right] =$$

$$\int_d^{+\infty} e^x \frac{1}{\sqrt{2\pi}\sqrt{\partial^2(\int_0^T \sigma^2(s)\,\mathrm{d}s + n\sigma_J^{\ 2}}} \exp\left\{-\frac{\left[x - n\partial\left(\ln(1+\theta) - \frac{1}{2}\sigma_J^{\ 2}\right)\right]^2}{2\partial^2\left(\int_0^T \sigma^2(s)\,\mathrm{d}s + n\sigma_J^{\ 2}\right)}\right\}\mathrm{d}x =$$

$$\exp\left\{n\partial\ln(1+\theta) + \frac{\partial^2}{2}\int_0^T \sigma^2(s) + \frac{n\partial(\partial-1)}{2}\sigma_J^{\ 2}\right\}\int_d^{+\infty} \frac{1}{\sqrt{2\pi}\sqrt{\partial^2\left(\int_0^T \sigma^2(s)\,\mathrm{d}s + n\sigma_J^{\ 2}\right)}} \cdot$$

$$\exp\left\{-\frac{\left[x - \left(n\partial\ln(1+\theta) + \partial^2\int_0^T \sigma^2(s)\,\mathrm{d}s + n\partial\left(\partial - \frac{1}{2}\right)\sigma_J^{\ 2}\right)\right]^2}{2\partial^2\left(\int_0^T \sigma^2(s)\,\mathrm{d}s + n\sigma_J^{\ 2}\right)}\right\}\mathrm{d}x$$

令

$$\frac{x - \left[n\partial\ln(1+\theta) + \partial^2\int_0^T \sigma^2(s)\,\mathrm{d}s + n\partial\left(\partial - \frac{1}{2}\right)\sigma_J^{\ 2}\right]}{\sqrt{2\partial^2\left(\int_0^T \sigma^2(s)\,\mathrm{d}s + n\sigma_J^{\ 2}\right)}} \triangleq y$$

则 $\mathrm{d}x = \sqrt{2\partial^2\left(\int_0^T \sigma^2(s)\,\mathrm{d}s + n\sigma_J^{\ 2}\right)}\,\mathrm{d}y$.

$$E[e^x I\{x > d\}] = \exp\left\{n\partial\ln(1+\theta) + \frac{\partial^2}{2}\int_0^T \sigma^2(s) + \frac{n\partial(\partial-1)}{2}\sigma_J^{\ 2}\right\} \cdot$$

$$\int_{\frac{d-\left[n\partial\ln(1+\theta)+\partial^2\int_0^T\sigma^2(s)\mathrm{d}s+n\partial(\partial-\frac{1}{2})\sigma_J^2\right]}{\sqrt{2\partial^2\left(\int_0^T\sigma^2(s)\mathrm{d}s+n\sigma_J^2\right)}}}^{+\infty}\frac{1}{\sqrt{2\pi}\sqrt{\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}}\cdot$$

$$\exp\left\{-\frac{y^2}{2}\right\}\sqrt{2\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}\mathrm{d}y=\exp\left\{n\partial\ln(1+\theta)+\frac{\partial^2}{2}\int_0^T\boldsymbol{\sigma}^2(s)+\frac{n\partial(\partial-1)}{2}\boldsymbol{\sigma}_J{}^2\right\}\cdot$$

$$N\left(-\frac{d-\left[n\partial\ln(1+\theta)+\partial^2\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\partial(\partial-\frac{1}{2})\boldsymbol{\sigma}_J{}^2\right]}{\sqrt{2\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}}\right)=$$

$$\exp\left\{n\partial\ln(1+\theta)+\frac{\partial^2}{2}\int_0^T\boldsymbol{\sigma}^2(s)+\frac{n\partial(\partial-1)}{2}\boldsymbol{\sigma}_J{}^2\right\}N(d_n)$$

且

$$d_n=\frac{\ln\frac{(1+\theta)^{n\partial}s^{\partial}}{k}+n\partial\left(\partial-\frac{1}{2}\right)\boldsymbol{\sigma}_J{}^2+\int_0^T\left[(\partial-1)\boldsymbol{\mu}(s)-\partial q(s)-\partial\boldsymbol{\lambda}(s)\theta+\frac{\partial(2\partial-1)}{2}\boldsymbol{\sigma}^2(s)+r(s)\right]\mathrm{d}s}{\sqrt{2\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}}$$

$$E[I\{x>d\}]=\int_d^{+\infty}\frac{1}{\sqrt{2\pi}\sqrt{\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}}\exp\left\{-\frac{\left[x-n\partial\left(\ln(1+\theta)+\frac{1}{2}\boldsymbol{\sigma}_J{}^2\right)\right]^2}{2\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}\right\}\mathrm{d}x$$

令 $\frac{x-n\partial\left(\ln(1+\theta)+\frac{1}{2}\boldsymbol{\sigma}_J{}^2\right)}{\sqrt{2\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}}\triangleq z$,则 $\mathrm{d}x=\sqrt{2\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}\mathrm{d}z$.

于是

$$E[I\{x>d\}]=\int_{\frac{d-\left(n\partial\ln(1+\theta)+\frac{1}{2}\sigma_J^2\right)}{\sqrt{2\partial^2\left(\int_0^T\sigma^2(s)\mathrm{d}s+n\sigma_J^2\right)}}}^{+\infty}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}}\exp\left\{-\frac{z^2}{2}\right\}\sqrt{2\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}\mathrm{d}z=$$

$$N\left(-\frac{d-n\partial(\ln(1+\theta)+\frac{1}{2}\boldsymbol{\sigma}_J{}^2)}{\sqrt{2\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}}\right)=N\left(d_n-\sqrt{\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}\right)$$

利用条件期望的性质并结合全期望公式,可以得到

$$E[e^xI\{x>d\}]=E[E[e^xI\{x>d\}]|N(T)]=$$

$$\sum_{i=0}^{+\infty}\frac{\exp\left\{-\int_0^T\boldsymbol{\lambda}(s)\mathrm{d}s\right\}\left(\int_0^T\boldsymbol{\lambda}(s)\mathrm{d}s\right)^n}{n!}\cdot(1+\theta)^{n\partial}\exp\left\{\frac{\partial^2}{2}\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+\frac{n\partial(\partial-1)}{2}\boldsymbol{\sigma}_J{}^2\right\}N(d_n)\triangleq\prod\nolimits_1$$

$$E[I\{x>d\}]=E[E[I\{x>d\}]|N(T)]=$$

$$\sum_{i=0}^{+\infty}\frac{\exp\left\{-\int_0^T\boldsymbol{\lambda}(s)\mathrm{d}s\right\}(\int_0^T\boldsymbol{\lambda}(s)\mathrm{d}s)^n}{n!}N\left(d_n-\sqrt{\partial^2\left(\int_0^T\boldsymbol{\sigma}^2(s)\mathrm{d}s+n\boldsymbol{\sigma}_J{}^2\right)}\right)\triangleq\prod\nolimits_2$$

则

$$C_1=E\left[\exp\left\{-\int_0^T\boldsymbol{\mu}(s)\mathrm{d}s\right\}S^{\partial}(T)I\left\{\exp\left\{-\int_0^T\boldsymbol{\mu}(s)\mathrm{d}s\right\}S^{\partial}(T)>\exp\{-\int_0^Tr(s)\mathrm{d}s\}k\right\}\right]=$$

$$s^{\partial}\exp\left\{\int_0^T\left[(\partial-1)\boldsymbol{\mu}(s)-\partial q(s)-\partial\boldsymbol{\lambda}(s)\theta-\frac{\partial}{2}\boldsymbol{\sigma}^2(s)\right]\mathrm{d}s\right\}E[e^xI\{x>d\}]=$$

$$s^{\partial}\exp\left\{\int_0^T\left[(\partial-1)\boldsymbol{\mu}(s)-\partial q(s)-\partial\boldsymbol{\lambda}(s)\theta-\frac{\partial(\partial-1)}{2}\boldsymbol{\sigma}^2(s)\right]\mathrm{d}s\right\}\cdot$$

$$\sum_{i=0}^{+\infty}\frac{\exp\left\{-\int_0^T\lambda(s)\mathrm{d}s\right\}\left(\int_0^T\lambda(s)\mathrm{d}s\right)^n}{n!}(1+\theta)^{n\partial}\exp\left\{\frac{n\partial(\partial-1)}{2}\sigma_J{}^2\right\}N(d_n)=$$

$$s^{\partial}\exp\left\{\int_0^T\left[(\partial-1)\mu(s)-\partial q(s)-\partial\lambda(s)\theta-\frac{\partial(\partial-1)}{2}\sigma^2(s)\right]\mathrm{d}s\right\}\cdot$$

$$\sum_{i=0}^{+\infty}\frac{\exp\left\{-\int_0^T(1+\theta)^{\partial}\lambda(s)\mathrm{d}s\right\}\left((1+\theta)^{\partial}\int_0^T\lambda(s)\mathrm{d}s\right)^n}{n!}\exp\left\{\frac{n\partial(\partial-1)}{2}\sigma_J{}^2\right\}N(d_n)$$

$$C_2=E\left[\exp\left\{-\int_0^T r(s)\mathrm{d}s\right\}k\cdot I\left\{\exp\left\{-\int_0^T\mu(s)\mathrm{d}s\right\}s^{\partial}(T)>\exp\left\{-\int_0^T r(s)\mathrm{d}s\right\}k\right\}\right]=$$

$$k\exp\left\{-\int_0^T r(s)\mathrm{d}s\right\}E[I\{x>d\}]=$$

$$k\exp\left\{-\int_0^T r(s)\mathrm{d}s\right\}\sum_{i=0}^{+\infty}\frac{\exp\left\{-\int_0^T\lambda(s)\mathrm{d}s\right\}\left(\int_0^{\mathrm{T}}\lambda(s)\mathrm{d}s\right)^n}{n!}N\left(d_n-\sqrt{\partial^2\left(\int_0^T\sigma^2(s)\mathrm{d}s+n\sigma_J{}^2\right)}\right)$$

由此可得

$$C(K,T)=s^{\partial}\exp\left\{\int_0^t\left[(\partial-1)\mu(s)-\partial q(s)-\partial\lambda(s)\theta-\frac{\partial(\partial-1)}{2}\sigma^2(s)\right]\mathrm{d}s\right\}\prod\nolimits_1-k\exp\left\{-\int_0^T r(s)\mathrm{d}s\right\}\prod\nolimits_2$$

定理 2　股票价格服从上述模型要求,欧式看跌幂期权的保险精算法定价公式为

$$P(K,T)=k\exp\left\{-\int_0^T r(s)\mathrm{d}s\right\}\prod\nolimits_2-s^{\partial}\exp\left\{\int_0^T\left[(\partial-1)\mu(s)-\partial q(s)-\partial\lambda(s)\theta-\frac{\partial(\partial-1)}{2}\sigma^2(s)\right]\mathrm{d}s\right\}\prod\nolimits_1$$

其中

$$\prod\nolimits_1=\sum_{i=0}^{+\infty}\frac{\exp\left\{-\int_0^T\lambda(s)\mathrm{d}s\right\}\left(\int_0^T\lambda(s)\mathrm{d}s\right)^n}{n!}\cdot(1+\theta)^{n\partial}\exp\left\{\frac{\partial^2}{2}\int_0^T\sigma^2(s)\mathrm{d}s+\frac{n\partial(\partial-1)}{2}\sigma_J{}^2\right\}N(d_n)$$

$$\prod\nolimits_2=\sum_{i=0}^{+\infty}\frac{\exp\left\{-\int_0^{\mathrm{T}}\lambda(s)\mathrm{d}s\right\}\left(\int_0^T\lambda(s)\mathrm{d}s\right)^n}{n!}N\left(d_n-\sqrt{\partial^2\left(\int_0^T\sigma^2(s)\mathrm{d}s+n\sigma_J{}^2\right)}\right)$$

$$d_n=\frac{\ln\frac{(1+\theta)^{n\partial}s^{\partial}}{k}+n\partial\left(\partial-\frac{1}{2}\right)\sigma_J{}^2+\int_0^T\left[(\partial-1)\mu(s)-\partial q(s)-\partial\lambda(s)\theta+\frac{\partial(2\partial-1)}{2}\sigma^2(s)+r(s)\right]\mathrm{d}s}{\sqrt{2\partial^2\left(\int_0^T\sigma^2(s)\mathrm{d}s+n\sigma_J{}^2\right)}}$$

证明　由定理 1 易得,略.

综合定理 1 和定理 2,可以得到保险精算法中带有连续红利支付的欧式幂期权的定价公式.

3　结语

假设股票价格过程服从带非齐次 Poisson 跳的扩散过程,本文利用保险精算定价方式给出带有连续支付红利的一类奇异期权——欧式幂期权——看涨和看跌两种情形下的定价公式. 关于红利的支付形式,还有很多研究空间,例如随机时间进行分红以及当股票价格达到一定阈值时进行分红. 不同的分红模式对投资者和经营者的选择会有不同的影响,寻求一种更为合理的分红机制是未来研究的方向.

参考文献:

[1]　Black F,Scholes M. The pricing of options and corporate liabilities[J]. Journal of Political Economy,1973,81(3): 637.

[2]　Merton R. Theory of rational option pricingn[J]. Bell Journal of Economics and Mnagement Science,1973,4(1):141.

[3]　Bladt M,Rydberg T H. An actuarial approach to option pricing under the physical measure and without market assumptions[J]. Mathematics and Economics,1998,22(1):65.

[4] 闫海峰,刘三阳.带有 Poisson 跳的股票价格模型的期权定价[J].工程数学学报,2003,20(2):35.
[5] 刘冬艳,张军芳.连续支付红利的跳扩散模型交换期权定价[J].数学理论与应用,2013,33(2):75.
[6] 唐仕冰,费为银,李帅.股票带有机制转换与红利支付的定性期权估值问题研究[J].数学理论与应用,2013,33(3):33.
[7] 吴金美,金治明,凌晓东.扩散市场模型中带交易费和红利的欧式未定权益的保值与定价[J].工程数学学报,2013,30(6):45.
[8] 崔立梅.欧式幂期权的保险精算法定价[D].乌鲁木齐:新疆大学,2008.
[9] 王嘉展,刘丽霞.随机利率下股票价格服从几何分数布朗运动的幂期权定价[J].河北师范大学学报:自然科学版,2014(3):35.

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